

$J/\psi \rightarrow DP, DV$ decays in the QCD factorization approach

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Abstract

Motivated by the recent measurements on nonleptonic J/ψ weak decays at BESIII and the potential prospects of J/ψ meson at the high-luminosity heavy-flavor experiments, the branching ratios of the two-body nonleptonic $J/\psi \rightarrow DP, DV$ decays are estimated quantitatively by considering the QCD radiative corrections to hadronic matrix elements with the QCD factorization approach. It is found that the Cabibbo favored $J/\psi \rightarrow D_s^- \rho^+, D_s^- \pi^+, \overline{D}_u^0 \overline{K}^{*0}$ decays have branching ratios $\gtrsim 10^{-10}$, which might be promisingly detectable in the near future.

PACS numbers: 13.25.Gv 12.39.St 14.40.Pq 14.65.Dw

Keywords: J/ψ meson; weak decay; branching ratio; QCD factorization

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I. INTRODUCTION

The $J^{PC} = 1^{--}$ ground state of the charmonium family, the J/ψ meson, was discovered in 1974 simultaneously both from the e^+e^- invariant mass by the MIT-BNL group [1] and from an enormous increase of the cross sections for hadronic, $\mu^+\mu^-$ and e^+e^- final states by the SLAC-LBL group [2]. Since then the study on the J/ψ particle and its family members has attracted much persistent attentions of experimentalists and theorists due to the facts that: on the one hand, the charmonium states offer an excellent platform to test and improve our understanding of the strong interactions at both perturbative and nonperturbative levels; on the other hand, there is a great renewed interest due to the massive dedicated investigation at BES, CLEO-c, LHCb and the studies via decays of B mesons at B factories.

One of the most surprising feature of the J/ψ meson is its narrow width, $\Gamma_{J/\psi} = 92.9 \pm 2.8$ keV [3], which indicates that the decays of J/ψ into light hadrons are suppressed dynamically. The reason for the extremely small decay width of the J/ψ meson is usually referred to by the phenomenological OZI (Okubo-Zweig-Iizuka) rules [4–6], which states that processes with “detached” quark lines are suppressed. It is well known that the mass of the J/ψ meson is below the $D\bar{D}$ threshold. Hence, in despite of the OZI suppression, the J/ψ decay into hadrons are dominated by the strong and electromagnetic interactions, and the decay modes at the lowest order approximation could be divided into four types: (1) the hadronic decay via the annihilation of the $c\bar{c}$ quark pairs into three gluons, *i.e.*, $J/\psi \rightarrow ggg \rightarrow X$, (2) the electromagnetic decay via the $c\bar{c}$ annihilation into a virtual photon, *i.e.*, $J/\psi \rightarrow \gamma^* \rightarrow X$, (3) the radiative decay via the $c\bar{c}$ annihilation into one photon and two gluons, *i.e.*, $J/\psi \rightarrow \gamma gg \rightarrow \gamma + X$, (4) the magnetic dipole transition to η_c , *i.e.*, $J/\psi \rightarrow \gamma \eta_c \rightarrow \gamma + X$ [7, 8], where X denotes the possible final hadrons. Besides, the J/ψ meson can decay into hadrons also via the weak interactions, although the branching ratio for inclusive weak decays via a single c or \bar{c} quark decay relying on the spectator model is very small, about $2/(\tau_D \Gamma_{J/\psi}) \sim 10^{-8}$ [3, 9]. In this paper, we will concentrate on the flavor-changing nonleptonic $J/\psi \rightarrow DM$ weak decays with the QCD factorization (QCDF) approach [10–15], where M denotes the low-lying $SU(3)$ pseudoscalar and vector meson nonet. The reasons are listed as follows.

From the experimental point of view, (1) with the running of high-luminosity dedicated heavy-flavor factories, more and more J/ψ events have been accumulating. It is hopefully expected to produce about 10^{10} J/ψ events at BESIII per year of data taking with the

designed luminosity [16], and over 10^{10} prompt J/ψ events at LHCb per fb^{-1} data [17]. The availability of such large samples enables a realistic possibility to explore experimentally the nonleptonic J/ψ weak decays, so the corresponding theoretical studies are very necessary to provide a ready reference. (2) The detection of a single D meson coming from the J/ψ weak process is free from inefficient double tagging of the charmed meson pairs occurring above the $D\bar{D}$ threshold. In addition, the definite energies and momenta of the back-to-back final states in the center-of-mass frame of the J/ψ meson would provide an unambiguous signature. With the help of remarkable improvements of experimental instrumentation and sophisticated particle identification techniques, the accurate measurements on the hadronic $J/\psi \rightarrow DM$ weak decays may now be feasible. Recently, a search for the Cabibbo favored $J/\psi \rightarrow D_s\rho, D_uK^*$ decays is performed with available 2.25×10^8 J/ψ events accumulated with the BESIII detector, but no evident signal is observed due to insufficient statistics [18]. Of course, such small branching ratios make the observation of nonleptonic J/ψ weak decays extremely challenging, and observation of an abnormally large production rate of single charmed mesons in e^+e^- collisions would be a hint of new physics beyond the standard model.

From the theoretical point of view, (1) the nonleptonic J/ψ weak decay has been studied in previous works using the factorization scheme, such as Ref.[9] based on the spin symmetry and nonrecoil approximation, Ref.[19] with the QCD sum rules, Ref.[20] with the covariant light-cone quark model, and Refs.[21–23] with the Bauer-Stech-Wirbel (BSW) model [24, 25]. Due to that the transition form factor is one of the essential ingredients for the charmonium weak decay, the previous studies [9, 19–23] mainly concern the calculation of the weak transition form factors dominated by the nonperturbative dynamics, which lead surely to unavoidable uncertainties on theoretical predictions. Since the charmonium could be well handled with the nonrelativistic QCD, observables of the $J/\psi \rightarrow DM$ decays might be used to test and ameliorate various models by comparison with measurements. (2) In recent years, several attractive QCD-inspired methods have been substantially developed and successfully used to cope with the hadronic matrix elements of nonleptonic B weak decays, such as the soft and collinear effective theory [26–33] and QCDF based on the collinear factorization approximation and power counterterm rules in the heavy quark limit, the perturbative QCD approach [34–39] based on the k_T factorization scheme. These methods mainly concern the underlying dynamical mechanism of the weak decays of heavy flavor hadrons, and could

be applied to the weak decays of heavy quarkonium. The analysis of nonleptonic J/ψ weak decays are particularly interesting in exploring mechanism responsible for hadronic transitions and very important for study of the applicability of factorization theorem and QCD properties at the scale of $\mathcal{O}(m_c)$. Further, the weak decay of the J/ψ particle offers a unique opportunity to probe polarization effects involved in vector meson decays, which might be helpful to investigate the underlying structure and dynamics of heavy quarkonium.

This paper is organized as follows. In section II, we will present the theoretical framework and the amplitudes for nonleptonic $J/\psi \rightarrow DM$ weak decays within the QCDF framework. The section III is devoted to numerical results and discussion. Finally, the section IV is our summation.

II. THEORETICAL FRAMEWORK

A. The effective Hamiltonian

The low energy effective Hamiltonian responsible for the nonleptonic $J/\psi \rightarrow DM$ weak decays can be written as [40]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q_1, q_2} V_{cq_1}^* V_{uq_2} \left\{ C_1(\mu) Q_1(\mu) + C_2(\mu) Q_2(\mu) \right\} + \text{h.c.}, \quad (1)$$

where the Fermi coupling constant $G_F \simeq 1.166 \times 10^{-5} \text{ GeV}^{-2}$ [3]; $V_{cq_1}^* V_{uq_2}$ is the Cabibbo-Kobayashi-Maskawa (CKM) factor and $q_{1,2} = d, s$; The Wilson coefficients $C_{1,2}(\mu)$ summarize the physical contributions above the scale of μ . The expressions of the local tree four-quark operators are

$$Q_1 = [\bar{q}_{1,\alpha} \gamma_\mu (1 - \gamma_5) c_\alpha] [\bar{u}_\beta \gamma^\mu (1 - \gamma_5) q_{2,\beta}], \quad (2)$$

$$Q_2 = [\bar{q}_{1,\alpha} \gamma_\mu (1 - \gamma_5) c_\beta] [\bar{u}_\beta \gamma^\mu (1 - \gamma_5) q_{2,\alpha}], \quad (3)$$

where α and β are color indices and the sum over repeated indices is understood.

Here, we would like to point out that (1) due to the large cancellation of the CKM factors $V_{cd}^* V_{ud} + V_{cs}^* V_{us} \sim \mathcal{O}(\lambda^5)$ where the Wolfenstein parameter $\lambda = \sin \theta_c = 0.22537(61)$ [3] and θ_c is the Cabibbo angle, the contributions of penguin and annihilation operators are strongly suppressed and could be safely neglected if the CP -violating asymmetries that are expected to be very tiny due to the small weak phase difference for c quark decay are

prescinded from the present consideration. (2) The Wilson coefficients C_i are calculable with the perturbation theory and have properly been evaluated to the next-to-leading order (NLO). Their values at the scale of $\mu \sim \mathcal{O}(m_c)$ can be obtained with the renormalization group (RG) equation [40],

$$\vec{C}(\mu) = U_4(\mu, m_b) M(m_b) U_5(m_b, m_W) \vec{C}(m_W), \quad (4)$$

where $U_f(\mu_f, \mu_i)$ is the RG evolution matrix transforming the Wilson coefficients from the scale μ_i to μ_f , and $M(\mu)$ is the quark threshold matching matrix. The explicit expressions of $U_f(\mu_f, \mu_i)$ and $M(\mu)$ can be found in Ref.[40]. The numerical values of LO and NLO $C_{1,2}$ in naive dimensional regularization scheme are listed in Table I. The values of NLO Wilson coefficients in Table I are consistent with those given by Refs.[25, 40, 41], where a trick with “effective” number of active flavors $f = 4.15$ rather than formula Eq.(4) is used by Ref.[40]. (3) To obtain the decay amplitudes and branching ratios, the remaining works are how to accurately evaluate the hadronic matrix elements where the local operators are sandwiched between the initial and final states, which is also the most intricate melody in dealing with the weak decay of heavy hadrons by now.

B. Hadronic matrix elements

Phenomenologically, the simplest treatment on hadronic matrix elements of a four fermion operator is the approximation by the product of the decay constants and the transition form factors based on the color transparency ansatz [42] and the naive factorization scheme (NF) [43, 44]. As well known, the NF’s defects are very obvious and displayed as the absence of the renormalization scale dependence, the strong phases and the nonfactorizable corrections from the hadronic matrix elements, which result in nonphysical decay amplitudes and the incapacity of prediction on CP -violating asymmetries. To remedy this situation, M. Beneke *et al.* [10, 11] proposed that the hadronic matrix elements could be written as the convolution integrals of hard scattering kernels and the light cone distribution amplitudes with the QCDF approach.

Using the QCDF master formula, the hadronic matrix elements for the $J/\psi \rightarrow DM$ decays could be expressed as :

$$\langle DM | Q_i | J/\psi \rangle = \sum_i F_i^{J \rightarrow D} \int dx H_i(x) \Phi_M(x) = \sum_i F_i^{J \rightarrow D} f_M \{1 + \frac{\alpha_s}{\pi} r + \cdots\}, \quad (5)$$

where $F_i^{J \rightarrow D}$ is the transition form factor and $\Phi_M(x)$ is the light cone distribution amplitude of the emitted meson M with the decay constant f_M , which are assumed to be dominated by nonperturbative contributions and taken as universal inputs.

Here, we would like to point out that (1) for the $J/\psi \rightarrow DM$ decay, the spectator quark is the almost on-shell charm (anti)quark. It is commonly thought that the virtuality of the gluon tied up with the heavy spectator quark is of order Λ_{QCD}^2 . The hard and soft contributions associated with the charmed spectator entangle with each other and cannot be separated properly. According to the basic idea of the QCDF approach [11], the physical form factors that could be obtained from lattice QCD or QCD sum rules are introduced as inputs, and the hard spectator scattering contributions that are power suppressed in the heavy quark limit disappeared from Eq.(5). (2) The hard scattering kernels $H_i(x)$, including the nonfactorizable vertex corrections, are computable order by order with the perturbation theory in principle. At the order α_s^0 , $H_i(x) = 1$, *i.e.*, the convolution integral of Eq.(5) results in a decay constant and one goes back to the simple NF scenario. At the order α_s and higher orders, the information of the renormalization scale dependence and strong phases hidden in hadronic matrix elements could be partly recuperated. Combined the nonfactorizable contributions with the Wilson coefficients, the scale independent effective coefficients at the order α_s can be obtained [12]:

$$a_1 = C_1^{\text{NLO}} + \frac{1}{N_c} C_2^{\text{NLO}} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_2^{\text{LO}} V, \quad (6)$$

$$a_2 = C_2^{\text{NLO}} + \frac{1}{N_c} C_1^{\text{NLO}} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_1^{\text{LO}} V. \quad (7)$$

The expression of vertex corrections could be written as [12]:

$$V = 6 \log\left(\frac{m_c^2}{\mu^2}\right) - 18 - \left(\frac{1}{2} + i3\pi\right) a_0^M + \left(\frac{11}{2} - i3\pi\right) a_1^M - \frac{21}{20} a_2^M + \dots, \quad (8)$$

with the twist-2 distribution amplitudes of pseudoscalar and longitudinally polarized vector meson in terms of Gegenbauer polynomials [45–47]:

$$\phi_M(x) = 6 x \bar{x} \sum_{n=0}^{\infty} a_n^M C_n^{3/2}(x - \bar{x}), \quad (9)$$

where $\bar{x} = 1 - x$; a_n^M is the Gegenbauer moment and $a_0^M \equiv 1$.

It is found that (1) for the coefficient a_1 , nonfactorizable vertex corrections can provide $\geq 10\%$ enhancement compared with the NF's value, and a small strong phase $\leq 5^\circ$. (2) for the

coefficient a_2 , contributions of vertex corrections assisted with the large Wilson coefficient C_1 are significant, and a relatively large strong phase $\sim -115^\circ$ is obtained. (3) the magnitude of $a_{1,2}$ agrees well with that from the fit on hadronic D weak decays [48, 49], but with more information on the strong phases.

C. Decay amplitude

Within the QCDF framework, the Lorentz-invariant amplitudes for $J/\psi \rightarrow DM$ decays can be expressed as:

$$\mathcal{A}(J/\psi \rightarrow DM) = \langle DM | \mathcal{H}_{\text{eff}} | J/\psi \rangle = \frac{G_F}{\sqrt{2}} V_{cq_1}^* V_{uq_2} a_i \langle M | J^\mu | 0 \rangle \langle D | J_\mu | J/\psi \rangle. \quad (10)$$

The matrix elements of current operators are defined as follows:

$$\langle P(p) | A_\mu | 0 \rangle = -i f_P p_\mu, \quad (11)$$

$$\langle V(p, \epsilon) | V_\mu | 0 \rangle = f_V m_V \epsilon_\mu^*, \quad (12)$$

where f_P and f_V are the decay constants of pseudoscalar and vector mesons, respectively; m_V and ϵ denote the mass and polarization of vector meson, respectively.

The transition form factors are defined as follows [19–25]:

$$\begin{aligned} & \langle D(p_2) | V_\mu - A_\mu | J/\psi(p_1, \epsilon) \rangle \\ &= -\epsilon_{\mu\nu\alpha\beta} \epsilon_J^\nu q^\alpha (p_1 + p_2)^\beta \frac{V^{J \rightarrow D}(q^2)}{m_J + m_D} - i \frac{2 m_J \epsilon_J \cdot q}{q^2} q_\mu A_0^{J \rightarrow D}(q^2) \\ & \quad - i \epsilon_{J,\mu} (m_J + m_D) A_1^{J \rightarrow D}(q^2) - i \frac{\epsilon_J \cdot q}{m_J + m_D} (p_1 + p_2)_\mu A_2^{J \rightarrow D}(q^2) \\ & \quad + i \frac{2 m_J \epsilon_J \cdot q}{q^2} q_\mu A_3^{J \rightarrow D}(q^2), \end{aligned} \quad (13)$$

where $q = p_1 - p_2$; and $A_0(0) = A_3(0)$ is required compulsorily to cancel singularities at the pole $q^2 = 0$. There is a relation among these form factors

$$2m_J A_3(q^2) = (m_J + m_D) A_1(q^2) + (m_J - m_D) A_2(q^2). \quad (14)$$

It is clearly seen that there are only three independent form factors, $A_{0,1}(0)$ and $V(0)$, at the pole $q^2 = 0$ for the hadronic $J/\psi \rightarrow DM$ decays. From the convolution integral expressions of form factors at zero momentum transfer in terms of participating meson wave functions given in Refs.[23–25], there is approximately a hierarchic relationship, *i.e.*,

$$V^{J \rightarrow D}(0) \approx 3A_1^{J \rightarrow D}(0), \quad A_1^{J \rightarrow D}(0) \geq A_0^{J \rightarrow D}(0), \quad (15)$$

which are also verified by the numbers of Table 1 in Ref.[23].

With the above definition, amplitudes for $J/\psi \rightarrow DP$, DV decay are explicitly listed in the Appendix A and B. Here, we would like to point out that (1) the amplitudes for $J/\psi \rightarrow DV$ decays are conventionally expressed by helicity amplitudes, which They are defined by the decomposition [50–52],

$$\begin{aligned}\mathcal{H}_\lambda &= \langle V|J^\mu|0\rangle\langle D|J_\mu|J/\psi\rangle \\ &= \epsilon_V^{*\mu}\epsilon_J^\nu\left\{a g_{\mu\nu} + \frac{b}{m_J m_V}(p_J + p_D)^\mu p_V^\nu + \frac{i c}{m_J m_V}\epsilon_{\mu\nu\alpha\beta}p_V^\alpha(p_J + p_D)^\beta\right\}.\end{aligned}\quad (16)$$

The relations among helicity amplitudes and invariant amplitudes a , b , c are

$$\mathcal{H}_0 = -a x - 2b(x^2 - 1), \quad (17)$$

$$\mathcal{H}_\pm = a \pm 2c\sqrt{x^2 - 1}, \quad (18)$$

where the expressions of a , b , c and x are

$$a = -i f_V m_V (m_J + m_D) A_1^{J \rightarrow D}(q^2), \quad (19)$$

$$b = -i f_V m_J m_V^2 \frac{A_2^{J \rightarrow D}(q^2)}{m_J + m_D}, \quad (20)$$

$$c = +i f_V m_J m_V^2 \frac{V^{J \rightarrow D}(q^2)}{m_J + m_D}, \quad (21)$$

$$x = \frac{p_J \cdot p_V}{m_J m_V} = \frac{m_J^2 - m_D^2 + m_V^2}{2 m_J m_V}. \quad (22)$$

There scalar amplitudes a , b , c describe the s , d , p wave contributions, respectively. Clearly, compared with the s wave amplitude, the p and d wave amplitudes are suppressed by a factor of m_V/m_J . (2) The light vector mesons are assumed ideally mixed, *i.e.*, the $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\phi = s\bar{s}$. As for the mixing of pseudoscalar η and η' meson, we will adopt the quark-flavor basis description proposed in Ref.[53], and neglect the contributions from possible gluonium and $c\bar{c}$ compositions, *i.e.*,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \quad (23)$$

where $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$, respectively; the mixing angle $\phi = (39.3 \pm 1.0)^\circ$ [53].

The mass relations between physical states (η and η') and flavor states (η_q and η_s) are

$$m_{\eta_q}^2 = m_\eta^2 \cos^2\phi + m_{\eta'}^2 \sin^2\phi - \frac{\sqrt{2}f_{\eta_s}}{f_{\eta_q}}(m_{\eta'}^2 - m_\eta^2) \cos\phi \sin\phi, \quad (24)$$

$$m_{\eta_s}^2 = m_\eta^2 \sin^2\phi + m_{\eta'}^2 \cos^2\phi - \frac{f_{\eta_q}}{\sqrt{2}f_{\eta_s}}(m_{\eta'}^2 - m_\eta^2) \cos\phi \sin\phi. \quad (25)$$

III. NUMERICAL RESULTS AND DISCUSSION

In the rest frame of J/ψ particle, branching ratio for nonleptonic J/ψ weak decays can be written as

$$\mathcal{B}(J/\psi \rightarrow DM) = \frac{1}{12\pi} \frac{p_{\text{cm}}}{m_J^2 \Gamma_J} |\mathcal{A}(J/\psi \rightarrow DM)|^2, \quad (26)$$

where the common momentum of final states is

$$p_{\text{cm}} = \frac{\sqrt{[m_J^2 - (m_D + m_M)^2][m_J^2 - (m_D - m_M)^2]}}{2m_J}. \quad (27)$$

The input parameters in our calculation, including the CKM Wolfenstein parameters, decay constants of mesons, Gegenbauer moments of distribution amplitudes in Eq.(9), are collected in Table II. If not specified explicitly, we will take their central values as the default inputs. As well known, the transition form factors are essential parameters in the QCDF master formula of Eq.(5), but the discrepancy among previous results on form factors with different models (see Table 1 of Ref.[23]) is still large. In this paper, we will use the mean values of the form factors given in Ref.[21] with additional uncertainties to offer an order of magnitude estimation, *i.e.*,

$$A_0^{J \rightarrow D}(0) = 0.50 \pm 0.1, \quad A_0^{J \rightarrow D_s}(0) = 0.55 \pm 0.1, \quad (28)$$

$$A_1^{J \rightarrow D}(0) = 0.55 \pm 0.1, \quad A_1^{J \rightarrow D_s}(0) = 0.65 \pm 0.1, \quad (29)$$

$$V^{J \rightarrow D}(0) = 1.50 \pm 0.3, \quad V^{J \rightarrow D_s}(0) = 1.50 \pm 0.3. \quad (30)$$

Our numerical results on the CP -averaged branching ratios for $J/\psi \rightarrow DP$, DV decays are displayed in Table III, where theoretical uncertainties of the last column come from the CKM parameters, the renormalization scale $\mu = (1 \pm 0.2)m_c$, decay constants and Gegenbauer moments, transition form factors, respectively. For comparison, the previous results [19, 22, 23] with coefficients $a_1 = 1.26$ and $a_2 = -0.51$ are also listed in the columns 3–7 of Table III, where numbers in the columns 3 and 4–7 are calculated with different form factors based on QCD sum rules and BSW model, respectively; numbers in the columns 4–7 correspond to the form factors given by the flavor dependent ω (“A” column), QCD inspired $\omega = \alpha_s \times m$ (“B” column), the universal $\omega = 4$ GeV (“C” column) and $\omega = 5$ GeV (“D” column), respectively; and ω , the average transverse quark momentum, is a parameter of the BSW wave functions. The following are some comments.

(1) There are some differences among the estimations (see the numbers in Table III) on branching ratios for hadronic $J/\psi \rightarrow DP, DV$ decays. (i) The discrepancies among previous works, although the same values of coefficients $a_{1,2}$ are taken, come mainly from different values of form factors. (ii) Considering the effects of nonfactorizable vertex corrections, the QCDF's predictions on branching ratios agree basically with previous results, at least with the same order magnitude. The QCDF's results are generally in line with the numbers in columns "C" and "D" within uncertainties, because the values of form factors in our calculation is close to the average values of form factors used in columns "C" and "D".

(2) There are some hierarchical structures. (i) According to the coefficients $a_{1,2}$ and the CKM factors, the $J/\psi \rightarrow DP, DV$ decays could be divided into different cases listed below.

Case	Coefficient	CKM factor	Branching ratio	Decay modes
1 a	a_1	$ V_{ud}V_{cs}^* \sim 1$	$\gtrsim 10^{-9}$	$D_s \rho$
			$\gtrsim 10^{-10}$	$D_s \pi$
1 b	a_1	$ V_{ud}V_{cd}^* , V_{us}V_{cs}^* \sim \lambda$	$\gtrsim 10^{-10}$	$D_s K^*, D_d \rho$
			$\gtrsim 10^{-11}$	$D_s K, D_d \pi$
1 c	a_1	$ V_{us}V_{cd}^* \sim \lambda^2$	$\gtrsim 10^{-12}$	$D_d K^*, D_d K$
2 a	a_2	$ V_{ud}V_{cs}^* \sim 1$	$\gtrsim 10^{-10}$	$D_u K^*$
			$\gtrsim 10^{-11}$	$D_u K$
2 b	a_2	$ V_{ud}V_{cd}^* , V_{us}V_{cs}^* \sim \lambda$	$\gtrsim 10^{-11}$	$D_u \phi$
			$\gtrsim 10^{-12}$	$D_u \rho, D_u \omega, D_u \pi, D_u \eta$
2 c	a_2	$ V_{us}V_{cd}^* \sim \lambda^2$	$\gtrsim 10^{-13}$	$D_u \bar{K}^*, D_u \bar{K}$

The extremely small branching ratios for $J/\psi \rightarrow D_u \eta'$ decays is mainly due to the cancellation of CKM factors between $V_{ud}V_{cd}^* \sim -\lambda$ and $V_{us}V_{cs}^* \sim +\lambda$ resulting in the destructive interferences between the amplitudes $\mathcal{A}(J/\psi \rightarrow D_u \eta_q)$ and $\mathcal{A}(J/\psi \rightarrow D_u \eta_s)$. (ii) Compared with the $J/\psi \rightarrow DV$ decays, the $J/\psi \rightarrow DP$ decays are suppressed dynamically by the orbital angular momentum of final states $L_{DP} > L_{DV}$. (iii) In addition, after a careful scrutiny of the QCDF's results, it is interestingly found that there is an approximative relationship among the branching ratios for decay modes with the same charmed final state, $\mathcal{B}(J/\psi \rightarrow DV) \approx 5 \mathcal{B}(J/\psi \rightarrow DP)$, where the pseudoscalar P and vector V mesons corresponds to each other in the $SU(3)$ $q\bar{q}$ assignments of light mesons with the quark model, such as, $\mathcal{B}(J/\psi \rightarrow D_s^- \rho^+) \approx 5 \mathcal{B}(J/\psi \rightarrow D_s^- \pi^+)$ and so on. Hence, the CKM-favored a_1 dominated

$J/\psi \rightarrow D_s^- \rho^+$ decay has the largest branching ratio, which should be sought for with high priority and firstly observed by experimental physicists.

(3) There are many uncertainties on the QCDF's results. (i) The first uncertainty from the CKM factors is small due to the high precision on Wolfenstein parameter λ with only 0.3% relative errors now[3]. The second uncertainty from the renormalization scale could, in principle, be reduced by the inclusion of higher order α_s corrections to hadronic matrix elements, for example, it has been showed [54, 55] that tree amplitudes incorporating with the NNLO vertex corrections are relatively less sensitive to the choice of scale than the NLO amplitudes. The largest uncertainty (the fourth uncertainty), $\sim 40\%$, comes from the transition form factors, which is expected to be cancelled from the relative ratio of branching ratios, such as,

$$R_1 = \frac{\mathcal{B}(J/\psi \rightarrow D_s^- K^+)}{\mathcal{B}(J/\psi \rightarrow D_s^- \pi^+)} \approx |V_{us}|^2 \frac{f_K^2}{f_\pi^2} \approx (5.66_{-0.03-0.00-0.10-0.00}^{+0.03+0.01+0.10+0.00})\%, \quad (31)$$

$$R_2 = \frac{\mathcal{B}(J/\psi \rightarrow D_d^- K^+)}{\mathcal{B}(J/\psi \rightarrow D_d^- \pi^+)} \approx |V_{us}|^2 \frac{f_K^2}{f_\pi^2} \approx (5.95_{-0.03-0.00-0.10-0.00}^{+0.03+0.01+0.11+0.00})\%. \quad (32)$$

(ii) Uncertainties from other factors, such as the contributions of higher order α_s corrections to hadronic matrix elements, q^2 dependence of form factors, the final state interactions and so on, which deserve the dedicated study, are not considered in this paper. So one should not be too serious about the absolute size of the QCDF's branching ratios for $J/\psi \rightarrow DM$ decays which just provide an order of magnitude estimation.

IV. SUMMARY

In this paper, we present a phenomenological study on the nonleptonic $J/\psi \rightarrow DP$, DV weak decays with the QCDF approach. Our attention was fixed on the nonfactorizable contributions to hadronic matrix elements, while the weak transition form factors are taken as nonperturbative parameters, which is different from previous works [9, 19–23]. The values of coefficients $a_{1,2}$ incorporating QCD radiative corrections agree well with those obtained from the fit on hadronic D weak decays [48, 49], which imply that the QCDF approach might be valid for the J/ψ weak decays. Then the branching ratios of the exclusive $J/\psi \rightarrow DP$, DV weak decays are estimated. It is found that the QCDF's predictions on branching ratios are rough due to large uncertainties from input parameters, especially from form factors. Despite this, we still can get some information about the $J/\psi \rightarrow DP$, DV decays. For

example, the Cabibbo favored $J/\psi \rightarrow D_s^- \rho^+$, $D_s^- \pi^+$, $\bar{D}_u^0 \bar{K}^{*0}$ decays have relatively large branching ratios compared with other decay modes, which are promisingly detected at the high-luminosity heavy-flavor experiments in the forthcoming years.

Acknowledgments

The work is supported by both the National Natural Science Foundation of China (Grant Nos. 11475055, 11275057, U1232101 and U1332103) and the Program for Science and Technology Innovation Talents in Universities of Henan Province (Grant No. 2012HASTIT030 and 14HASTIT036). Q. Chang is also supported by the Foundation for the Author of National Excellent Doctoral Dissertation of China (Grant No. 201317).

Appendix A: The amplitudes for $J/\psi \rightarrow DP$ decays

$$\mathcal{A}(J/\psi \rightarrow D_s^- \pi^+) = \sqrt{2} G_F m_J (\epsilon_J \cdot p_\pi) f_\pi A_0^{J \rightarrow D_s} V_{cs}^* V_{ud} a_1, \quad (\text{A1})$$

$$\mathcal{A}(J/\psi \rightarrow D_s^- K^+) = \sqrt{2} G_F m_J (\epsilon_J \cdot p_K) f_K A_0^{J \rightarrow D_s} V_{cs}^* V_{us} a_1, \quad (\text{A2})$$

$$\mathcal{A}(J/\psi \rightarrow D_d^- \pi^+) = \sqrt{2} G_F m_J (\epsilon_J \cdot p_\pi) f_\pi A_0^{J \rightarrow D_d} V_{cd}^* V_{ud} a_1, \quad (\text{A3})$$

$$\mathcal{A}(J/\psi \rightarrow D_d^- K^+) = \sqrt{2} G_F m_J (\epsilon_J \cdot p_K) f_K A_0^{J \rightarrow D_d} V_{cd}^* V_{us} a_1, \quad (\text{A4})$$

$$\mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \pi^0) = -G_F m_J (\epsilon_J \cdot p_\pi) f_\pi A_0^{J \rightarrow D_u} V_{cd}^* V_{ud} a_2, \quad (\text{A5})$$

$$\mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 K^0) = \sqrt{2} G_F m_J (\epsilon_J \cdot p_K) f_K A_0^{J \rightarrow D_u} V_{cd}^* V_{us} a_2, \quad (\text{A6})$$

$$\mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \bar{K}^0) = \sqrt{2} G_F m_J (\epsilon_J \cdot p_K) f_K A_0^{J \rightarrow D_u} V_{cs}^* V_{ud} a_2, \quad (\text{A7})$$

$$\mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \eta_q) = G_F m_J (\epsilon_J \cdot p_{\eta_q}) f_{\eta_q} A_0^{J \rightarrow D_u} V_{cd}^* V_{ud} a_2, \quad (\text{A8})$$

$$\mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \eta_s) = \sqrt{2} G_F m_J (\epsilon_J \cdot p_{\eta_s}) f_{\eta_s} A_0^{J \rightarrow D_u} V_{cs}^* V_{us} a_2, \quad (\text{A9})$$

$$\mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \eta) = \cos\phi \mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \eta_q) - \sin\phi \mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \eta_s), \quad (\text{A10})$$

$$\mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \eta') = \sin\phi \mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \eta_q) + \cos\phi \mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \eta_s). \quad (\text{A11})$$

Appendix B: The amplitudes for $J/\psi \rightarrow DV$ decays

$$\begin{aligned} \mathcal{A}(J/\psi \rightarrow D_s^- \rho^+) &= -i \frac{G_F}{\sqrt{2}} f_\rho m_\rho V_{cs}^* V_{ud} a_1 \left\{ (\epsilon_\rho^* \cdot \epsilon_J) (m_J + m_{D_s}) A_1^{J \rightarrow D_s} \right. \\ &\quad \left. + (\epsilon_\rho^* \cdot p_J) (\epsilon_J \cdot p_\rho) \frac{2 A_2^{J \rightarrow D_s}}{m_J + m_{D_s}} - i \epsilon_{\mu\nu\alpha\beta} \epsilon_\rho^{*\mu} \epsilon_J^\nu p_\rho^\alpha p_J^\beta \frac{2 V^{J \rightarrow D_s}}{m_J + m_{D_s}} \right\}, \end{aligned} \quad (B1)$$

$$\begin{aligned} \mathcal{A}(J/\psi \rightarrow D_s^- K^{*+}) &= -i \frac{G_F}{\sqrt{2}} f_{K^*} m_{K^*} V_{cs}^* V_{us} a_1 \left\{ (\epsilon_{K^*}^* \cdot \epsilon_J) (m_J + m_{D_s}) A_1^{J \rightarrow D_s} \right. \\ &\quad \left. + (\epsilon_{K^*}^* \cdot p_J) (\epsilon_J \cdot p_{K^*}) \frac{2 A_2^{J \rightarrow D_s}}{m_J + m_{D_s}} - i \epsilon_{\mu\nu\alpha\beta} \epsilon_{K^*}^{*\mu} \epsilon_J^\nu p_{K^*}^\alpha p_J^\beta \frac{2 V^{J \rightarrow D_s}}{m_J + m_{D_s}} \right\}, \end{aligned} \quad (B2)$$

$$\begin{aligned} \mathcal{A}(J/\psi \rightarrow D_d^- \rho^+) &= -i \frac{G_F}{\sqrt{2}} f_\rho m_\rho V_{cd}^* V_{ud} a_1 \left\{ (\epsilon_\rho^* \cdot \epsilon_J) (m_J + m_{D_d}) A_1^{J \rightarrow D_d} \right. \\ &\quad \left. + (\epsilon_\rho^* \cdot p_J) (\epsilon_J \cdot p_\rho) \frac{2 A_2^{J \rightarrow D_d}}{m_J + m_{D_d}} - i \epsilon_{\mu\nu\alpha\beta} \epsilon_\rho^{*\mu} \epsilon_J^\nu p_\rho^\alpha p_J^\beta \frac{2 V^{J \rightarrow D_d}}{m_J + m_{D_d}} \right\}, \end{aligned} \quad (B3)$$

$$\begin{aligned} \mathcal{A}(J/\psi \rightarrow D_d^- K^{*+}) &= -i \frac{G_F}{\sqrt{2}} f_{K^*} m_{K^*} V_{cd}^* V_{us} a_1 \left\{ (\epsilon_{K^*}^* \cdot \epsilon_J) (m_J + m_{D_d}) A_1^{J \rightarrow D_d} \right. \\ &\quad \left. + (\epsilon_{K^*}^* \cdot p_J) (\epsilon_J \cdot p_{K^*}) \frac{2 A_2^{J \rightarrow D_d}}{m_J + m_{D_d}} - i \epsilon_{\mu\nu\alpha\beta} \epsilon_{K^*}^{*\mu} \epsilon_J^\nu p_{K^*}^\alpha p_J^\beta \frac{2 V^{J \rightarrow D_d}}{m_J + m_{D_d}} \right\}, \end{aligned} \quad (B4)$$

$$\begin{aligned} \mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \rho^0) &= +i \frac{G_F}{2} f_\rho m_\rho V_{cd}^* V_{ud} a_2 \left\{ (\epsilon_\rho^* \cdot \epsilon_J) (m_J + m_{D_u}) A_1^{J \rightarrow D_u} \right. \\ &\quad \left. + (\epsilon_\rho^* \cdot p_J) (\epsilon_J \cdot p_\rho) \frac{2 A_2^{J \rightarrow D_u}}{m_J + m_{D_u}} - i \epsilon_{\mu\nu\alpha\beta} \epsilon_\rho^{*\mu} \epsilon_J^\nu p_\rho^\alpha p_J^\beta \frac{2 V^{J \rightarrow D_u}}{m_J + m_{D_u}} \right\}, \end{aligned} \quad (B5)$$

$$\begin{aligned} \mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \omega) &= -i \frac{G_F}{2} f_\omega m_\omega V_{cd}^* V_{ud} a_2 \left\{ (\epsilon_\omega^* \cdot \epsilon_J) (m_J + m_{D_u}) A_1^{J \rightarrow D_u} \right. \\ &\quad \left. + (\epsilon_\omega^* \cdot p_J) (\epsilon_J \cdot p_\omega) \frac{2 A_2^{J \rightarrow D_u}}{m_J + m_{D_u}} - i \epsilon_{\mu\nu\alpha\beta} \epsilon_\omega^{*\mu} \epsilon_J^\nu p_\omega^\alpha p_J^\beta \frac{2 V^{J \rightarrow D_u}}{m_J + m_{D_u}} \right\}, \end{aligned} \quad (B6)$$

$$\begin{aligned} \mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \phi) &= -i \frac{G_F}{\sqrt{2}} f_\phi m_\phi V_{cs}^* V_{us} a_2 \left\{ (\epsilon_\phi^* \cdot \epsilon_J) (m_J + m_{D_u}) A_1^{J \rightarrow D_u} \right. \\ &\quad \left. + (\epsilon_\phi^* \cdot p_J) (\epsilon_J \cdot p_\phi) \frac{2 A_2^{J \rightarrow D_u}}{m_J + m_{D_u}} - i \epsilon_{\mu\nu\alpha\beta} \epsilon_\phi^{*\mu} \epsilon_J^\nu p_\phi^\alpha p_J^\beta \frac{2 V^{J \rightarrow D_u}}{m_J + m_{D_u}} \right\}, \end{aligned} \quad (B7)$$

$$\begin{aligned} \mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 K^{*0}) &= -i \frac{G_F}{\sqrt{2}} f_{K^*} m_{K^*} V_{cd}^* V_{us} a_2 \left\{ (\epsilon_{K^*}^* \cdot \epsilon_J) (m_J + m_{D_u}) A_1^{J \rightarrow D_u} \right. \\ &\quad \left. + (\epsilon_{K^*}^* \cdot p_J) (\epsilon_J \cdot p_{K^*}) \frac{2 A_2^{J \rightarrow D_u}}{m_J + m_{D_u}} - i \epsilon_{\mu\nu\alpha\beta} \epsilon_{K^*}^{*\mu} \epsilon_J^\nu p_{K^*}^\alpha p_J^\beta \frac{2 V^{J \rightarrow D_u}}{m_J + m_{D_u}} \right\}, \end{aligned} \quad (B8)$$

$$\begin{aligned} \mathcal{A}(J/\psi \rightarrow \bar{D}_u^0 \bar{K}^{*0}) &= -i \frac{G_F}{\sqrt{2}} f_{K^*} m_{K^*} V_{cs}^* V_{ud} a_2 \left\{ (\epsilon_{K^*}^* \cdot \epsilon_J) (m_J + m_{D_u}) A_1^{J \rightarrow D_u} \right. \\ &\quad \left. + (\epsilon_{K^*}^* \cdot p_J) (\epsilon_J \cdot p_{K^*}) \frac{2 A_2^{J \rightarrow D_u}}{m_J + m_{D_u}} - i \epsilon_{\mu\nu\alpha\beta} \epsilon_{K^*}^{*\mu} \epsilon_J^\nu p_{K^*}^\alpha p_J^\beta \frac{2 V^{J \rightarrow D_u}}{m_J + m_{D_u}} \right\}. \end{aligned} \quad (B9)$$

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TABLE I: The numerical values of the Wilson coefficients $C_{1,2}$ and effective coefficients $a_{1,2}$ for the $J/\psi \rightarrow D\pi$ decay at different scales, where $m_c = 1.275$ GeV.

μ	LO		NLO		NF		QCDF			
	C_1	C_2	C_1	C_2	a_1	a_2	$\text{Re}(a_1)$	$\text{Im}(a_1)$	$\text{Re}(a_2)$	$\text{Im}(a_2)$
$0.8 m_c$	1.335	-0.589	1.275	-0.504	1.107	-0.079	1.271	0.097	-0.453	-0.219
m_c	1.276	-0.505	1.222	-0.425	1.080	-0.018	1.217	0.069	-0.363	-0.173
$1.2 m_c$	1.239	-0.450	1.190	-0.374	1.065	0.022	1.185	0.054	-0.308	-0.149

TABLE II: The values of the Wolfenstein parameters, decay constants and Gegenbauer moments.

Wolfenstein parameters	
$\lambda = 0.22537 \pm 0.00061$ [3]	$A = 0.814^{+0.023}_{-0.024}$ [3]
$\bar{\rho} = 0.117 \pm 0.021$ [3]	$\bar{\eta} = 0.353 \pm 0.013$ [3]
decay constants	
$f_\pi = 130.41 \pm 0.20$ MeV [3]	$f_K = 156.2 \pm 0.7$ MeV [3]
$f_{\eta_q} = (1.07 \pm 0.02)f_\pi$ [53]	$f_{\eta_s} = (1.34 \pm 0.06)f_\pi$ [53]
$f_\rho = 216 \pm 3$ MeV [47]	$f_\omega = 187 \pm 5$ MeV [47]
$f_\phi = 215 \pm 5$ MeV [47]	$f_{K^*} = 220 \pm 5$ MeV [47]
Gegenbauer moments at the scale $\mu = 1$ GeV	
$a_1^\pi = a_1^{\eta_q} = a_1^{\eta_s} = 0$ [46]	$a_2^\pi = a_2^{\eta_q} = a_2^{\eta_s} = 0.25 \pm 0.15$ [46]
$a_1^{\bar{K}} = -a_1^K = 0.06 \pm 0.03$ [46]	$a_2^K = a_2^{\bar{K}} = 0.25 \pm 0.15$ [46]
$a_1^\rho = a_1^\omega = a_1^\phi = 0$ [47]	$a_2^\rho = a_2^\omega = 0.15 \pm 0.07$ [47]
$a_1^{\bar{K}^*} = -a_1^{K^*} = 0.03 \pm 0.02$ [47]	$a_2^{K^*} = a_2^{\bar{K}^*} = 0.11 \pm 0.09$ [47]
	$a_2^\phi = 0.18 \pm 0.08$ [47]

TABLE III: The branching ratios for $J/\psi \rightarrow DP, DV$ decay, The numbers in columns of Refs.[19, 22, 23] are calculated with coefficients $a_1 = 1.26$ and $a_2 = -0.51$. The results of Ref.[19] are based on QCD sum rules. The numbers in columns of “A”, “B”, “C” and “D” are based on BSW model with flavor dependent ω , QCD inspired $\omega = \alpha_s \times m$, universal $\omega = 0.4$ GeV and 0.5GeV , respectively. The uncertainties of the “QCDF” column come from the CKM parameters, the renormalization scale $\mu = (1\pm 0.2)m_c$, decay constants and Gegenbauer moments, form factors, respectively.

Decay modes	Case	Ref.[19]	Ref.[23]			Ref.[22]	This work QCDF
			A	B	C		
$D_s^- \pi^+$	1 a	2.0×10^{-10}	7.41×10^{-10}	7.13×10^{-10}	3.32×10^{-10}	8.74×10^{-10}	$(4.10^{+0.00+0.39+0.02+1.63}_{-0.00-0.22-0.02-1.35}) \times 10^{-10}$
$D_s^- K^+$	1 b	1.6×10^{-11}	5.3×10^{-11}	5.2×10^{-11}	2.4×10^{-11}	5.5×10^{-11}	$(2.32^{+0.01+0.22+0.03+0.92}_{-0.01-0.12-0.03-0.77}) \times 10^{-11}$
$D_d^- \pi^+$	1 b	0.8×10^{-11}	2.9×10^{-11}	2.8×10^{-11}	1.5×10^{-11}	5.5×10^{-11}	$(2.21^{+0.01+0.21+0.01+0.97}_{-0.01-0.12-0.01-0.79}) \times 10^{-11}$
$D_d^- K^+$	1 c	—	2.3×10^{-12}	2.2×10^{-12}	1.2×10^{-12}	—	$(1.31^{+0.01+0.13+0.02+0.58}_{-0.01-0.07-0.02-0.47}) \times 10^{-12}$
$\overline{D}_u^0 \pi^0$	2 b	—	2.4×10^{-12}	2.3×10^{-12}	1.2×10^{-12}	5.5×10^{-12}	$(1.21^{+0.01+0.69+0.02+0.53}_{-0.01-0.34-0.02-0.44}) \times 10^{-12}$
$\overline{D}_u^0 K^0$	2 c	—	4.0×10^{-13}	4.0×10^{-13}	2.0×10^{-13}	—	$(1.44^{+0.02+0.81+0.04+0.63}_{-0.02-0.40-0.04-0.52}) \times 10^{-13}$
$\overline{D}_u^0 \overline{K}^0$	2 a	3.6×10^{-11}	1.39×10^{-10}	1.34×10^{-10}	7.2×10^{-11}	2.8×10^{-10}	$(4.98^{+0.00+2.81+0.12+2.19}_{-0.00-1.38-0.11-1.79}) \times 10^{-11}$
$\overline{D}_u^0 \eta$	2 b	—	7.0×10^{-12}	6.7×10^{-12}	3.6×10^{-12}	1.6×10^{-12}	$(3.56^{+0.02+2.01+0.24+1.57}_{-0.02-0.99-0.29-1.28}) \times 10^{-12}$
$\overline{D}_u^0 \eta'$	—	—	4.0×10^{-13}	4.0×10^{-13}	2.0×10^{-13}	3.0×10^{-13}	$(2.02^{+0.01+1.14+0.23+0.89}_{-0.01-0.56-0.41-0.73}) \times 10^{-13}$
$D_s^- \rho^+$	1 a	1.26×10^{-9}	5.11×10^{-9}	5.32×10^{-9}	1.77×10^{-9}	3.63×10^{-9}	$(2.21^{+0.00+0.21+0.06+0.78}_{-0.00-0.12-0.06-0.66}) \times 10^{-9}$
$D_s^- K^{*+}$	1 b	0.82×10^{-10}	2.82×10^{-10}	2.96×10^{-10}	0.97×10^{-10}	2.12×10^{-10}	$(1.22^{+0.01+0.11+0.06+0.42}_{-0.01-0.06-0.06-0.36}) \times 10^{-10}$
$D_d^- \rho^+$	1 b	0.42×10^{-10}	2.16×10^{-10}	2.28×10^{-10}	0.72×10^{-10}	2.20×10^{-10}	$(1.09^{+0.01+0.10+0.03+0.45}_{-0.01-0.06-0.03-0.37}) \times 10^{-10}$
$D_d^- K^{*+}$	1 c	—	1.3×10^{-11}	1.3×10^{-11}	4.2×10^{-12}	—	$(6.14^{+0.07+0.58+0.30+2.51}_{-0.07-0.32-0.29-2.08}) \times 10^{-12}$
$\overline{D}_u^0 \rho^0$	2 b	—	1.8×10^{-11}	1.9×10^{-11}	6.0×10^{-12}	2.2×10^{-11}	$(5.93^{+0.03+3.35+0.20+2.45}_{-0.03-1.64-0.20-2.03}) \times 10^{-12}$
$\overline{D}_u^0 \omega$	2 b	—	1.6×10^{-11}	1.7×10^{-11}	5.0×10^{-12}	1.8×10^{-11}	$(4.45^{+0.02+2.51+0.27+1.84}_{-0.02-1.23-0.26-1.52}) \times 10^{-12}$
$\overline{D}_u^0 \phi$	2 b	—	4.2×10^{-11}	4.4×10^{-11}	1.4×10^{-11}	6.5×10^{-11}	$(1.11^{+0.01+0.63+0.06+0.45}_{-0.01-0.31-0.06-0.37}) \times 10^{-11}$
$\overline{D}_u^0 K^{*0}$	2 c	—	2.1×10^{-12}	2.2×10^{-12}	7.0×10^{-13}	—	$(6.69^{+0.07+3.78+0.38+2.74}_{-0.07-1.85-0.36-2.27}) \times 10^{-13}$
$\overline{D}_u^0 \overline{K}^{*0}$	2 a	1.54×10^{-10}	7.61×10^{-10}	8.12×10^{-10}	2.51×10^{-10}	1.03×10^{-9}	$(2.32^{+0.00+1.31+0.13+0.95}_{-0.00-0.64-0.12-0.79}) \times 10^{-10}$